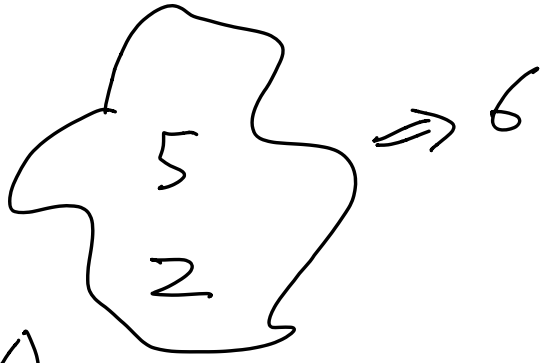
HEAP ("BINARY HEAP")

IS MOST IMPORTANT ~~[illegible]~~ ALGORITHM/DS FOR PRIORITY QUEUES.

INTERFACE

```
PUBLIC INTERFACE INTEGERMAXQUEUE {
    PUBLIC VOID INSERT(INT K);
    PUBLIC INT GETMAX();
    ... SIZE, ISEMPTY...
}
```

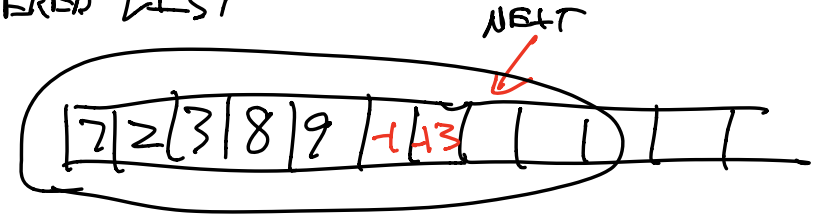
```
INSERT(5);
  "  (6);
  "  (2)
INT N = GETMAX() ⇒ 6
```



FIRST ATTEMPT

UNORDERED LIST

⇒ 12



N = # ELEMENTS IN ARRAY.
NOT LENGTH OF ARRAY

O(1) IN N — INSERT: ADD TO END

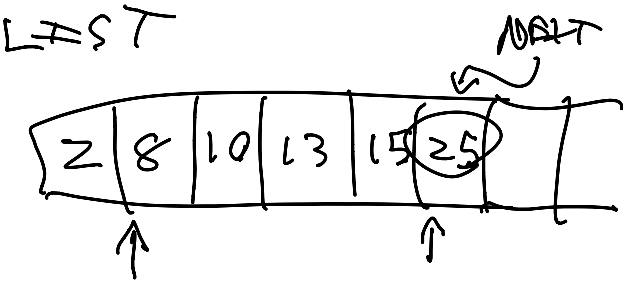
A[NEXT++] = K

O(N) — GETMAX: FIND MAX ELT
& DELETE
& MOVE LATER ONE OVER

O(N)

2ND ORDERED LIST

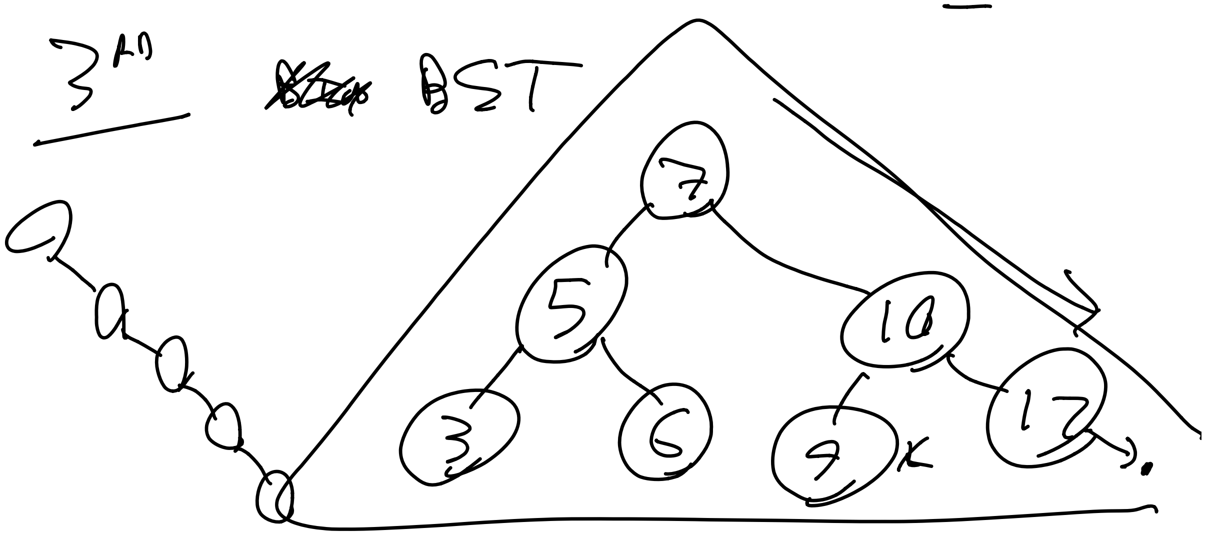
O(N) INSERT



O(1) GETMAX

RETURN A[--NEXT]

3RD BST



INSERT $O(N)$

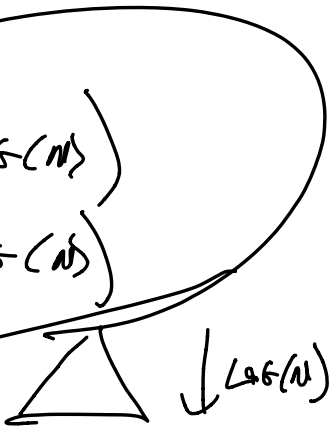
GETMAX $O(N)$

] WORST CASE

4TH 2-3 TREE

INSERT $O(\log(N))$

GETMAX $O(\log(N))$



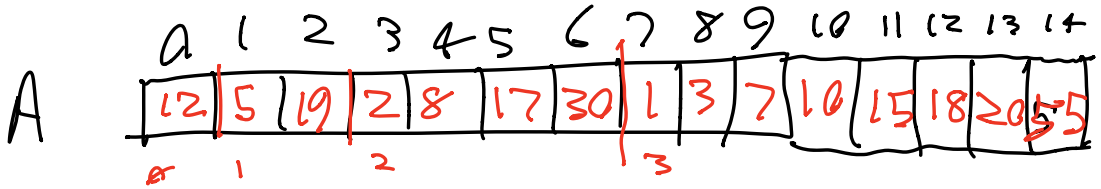
PUNCHLINE: WE WILL COMBINE BEST FEATURES OF 2-4 ~~AND~~ OF THESE

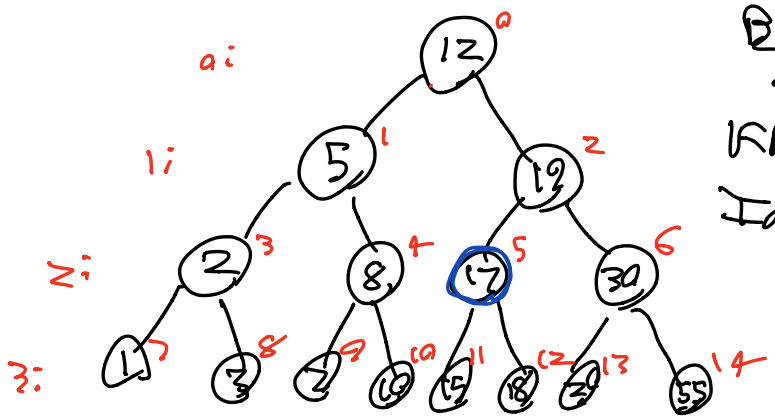
HEAP DATA STRUCTURE

* ARRAY BASED (NO EXPLICIT PTRS)
  ORDERED
  → TREE (BALANCED)

$O(\log(N))$ FOR ~~INSERT & GETMAX~~ INSERT & GETMAX

STORING A BINARY TREE IN AN ARRAY.





BASIC IDEA

STORE KEYS IN TREE IN LEVEL ORDER

WE WILL REFER TO ITEMS BY INDEX I IN A

I = 5

A[I] ⇒ 17

// RETURN INDEX OF PARENT OR -1 IF AT ROOT

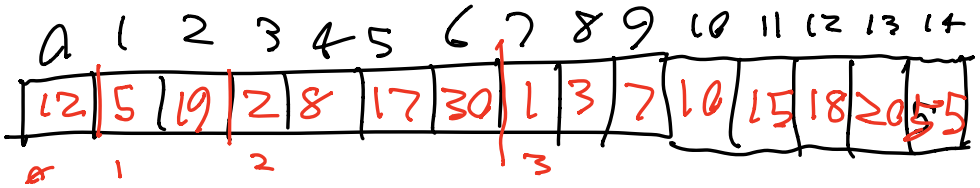
PARENT(5) ⇒ 2

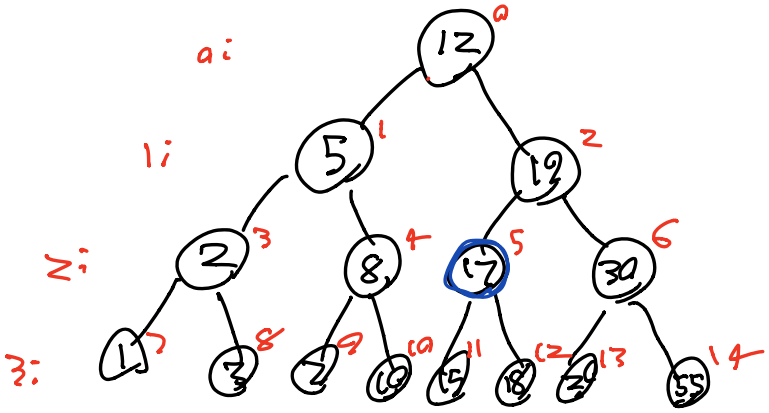
```
INT PARENT(INT I) {
    RETURN (I-1)//2;
}
```

(-1)//2

i	PARENT(i)	i//2	(i-1)//2
1	0	0	0
2	0	1	0
3	1	1	
4	1	2	
5	2	2	
6	2	3	
7	3	3	
8	3	4	
9	4	4	
10	4		
11	5		
12	5		

A

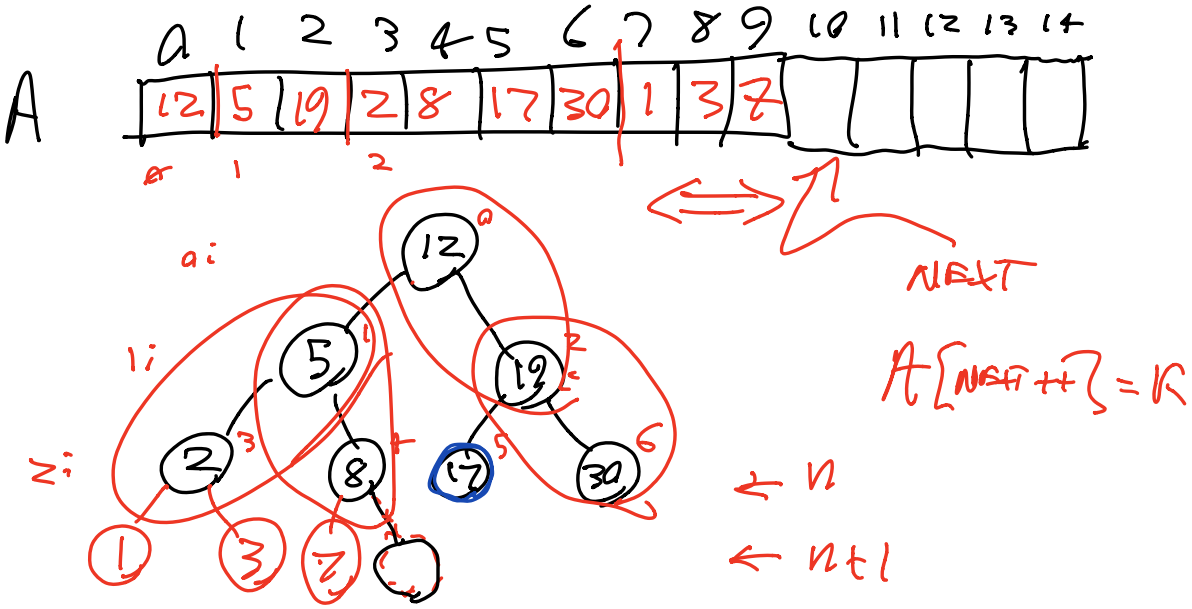





```
INT LCHILD(INT I) {
    RETURN 2*I+1
}

INT RCHILD(INT I) {
    RETURN 2*I+2
}
```

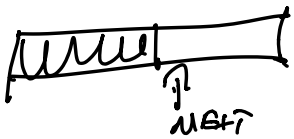
I	LC	RC	2*I
0	1	2	0
1	3	4	2
2	5	6	4
3	7	8	6
4	9	10	8
5	11	12	10



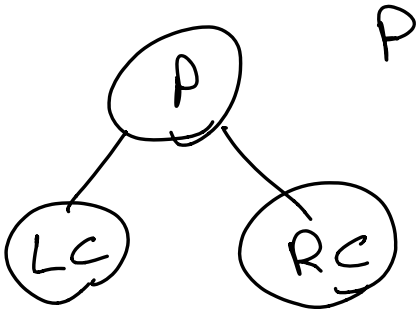
IF TREE IS "BALANCED FROM LEFT" IF BALANCED AND ALL LEAVES AT LEVEL n ARE TO RIGHT OF LEAVES AT LEVEL n+1.

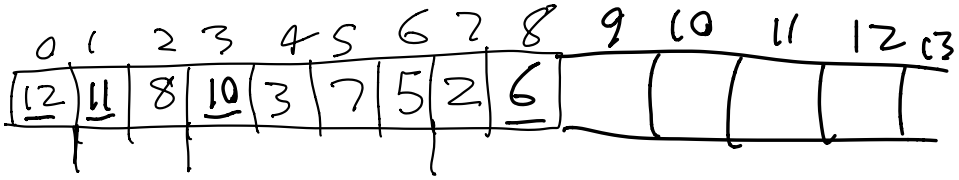
FILL IN NODES IN LEVEL ORDER.

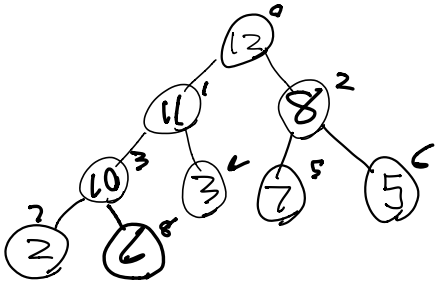
A HEAP IS AN ARRAY IN PREVIOUS FORM AND HAS "HEAP ORDERING PROPERTY" (ASSUME MAX Q)



NAMELY: FOR ANY PARENT P, $P \geq LC$ & $P \geq RC$







INSERT: PUT NEW KEY IN NEXT LEVEL ORDER POSITION

INSERT NEW KEY

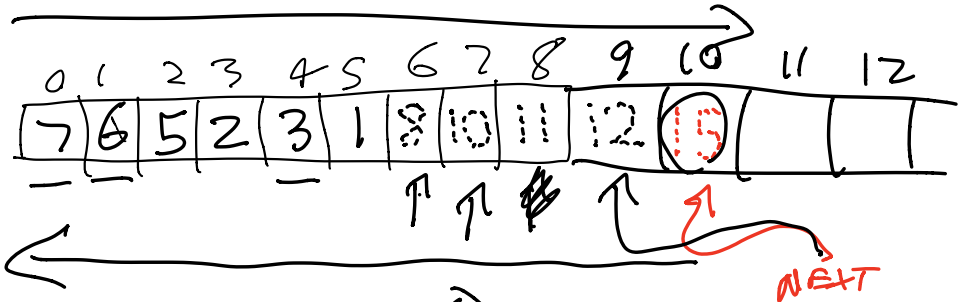
10
6

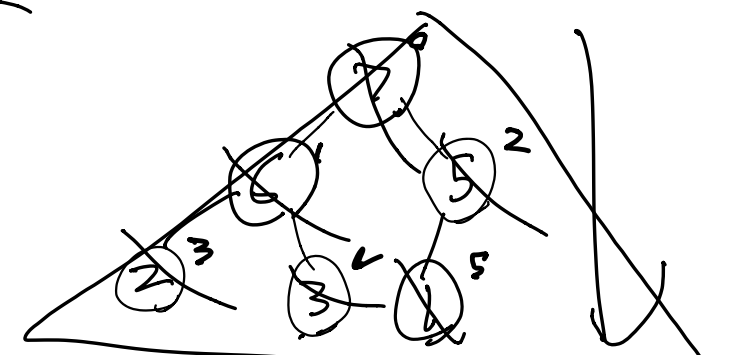
IN ORDER IN PATH TO ROOT

JUST AS IN INSERTION SORT

COMPARE WITH PARENT & SWAP IF LARGER THAN PARENT.

N == NEXT





--NEXT;

BALANCED!

~~INSERT~~ & GETMAX ARE O(LOG(N))

USES ARRAY (CAN RESIZE)

DELETE MAX

SWAP ROOT WITH LAST KEY

SWAP NEW ROOT WITH MAXIMAL CHILD UNTIL IN HEAP ORDER.

RETURN A[NEXT]